

12/12/17 "Quality is not an act, it is a habit" -Aristotle

HW: Completing the Square worksheet (finish up to #10)

Test 2 on Wednesday 12/20

AIM: How do we solve quadratic equations by Completing the Square?

Warm Up: Factor the following:

1) $x^2 + 2x + 1$
 $(x+1)(x+1) = (x+1)^2$

2) $x^2 - 4x + 4$
 $(x-2)(x-2)$ or $(x-2)^2$

3) $x^2 + 6x + 9$
 $(x+3)(x+3)$ or $(x+3)^2$

4) $x^2 - 8x + 16$
 $(x-4)^2$

⊛ These trinomials are all considered perfect square trinomials.

How can we solve a quadratic equation if we can not factor?

-We use a process called Completing the Square

5) What are the roots of: $x^2 + 8x - 4 = 0$

1) The "a" must be 1 (If it isn't, then divide the entire equation by the "a" value)	$x^2 + 8x - 4 = 0$
2) Move the "c" (constant) term to the right side	$x^2 + 8x = 4$
3) Insert a space on each side in order to create a perfect square trinomial (remember to do it to both sides)	$x^2 + 8x + \square = 4 + \square$
4) Divide the "b" by 2 and then square the result. Add the result to both sides of the equation <div style="display: flex; align-items: center; justify-content: center;"> $x^2 + 8x + \square = 4 + \square$ <div style="margin-left: 20px;"> <p style="color: red; font-size: 1.2em;">half</p> <p style="color: red; font-size: 1.2em;">square</p> <p style="color: red; font-size: 1.2em;">share</p> </div> </div>	$x^2 + 8x + 16 = 4 + 16$ <div style="text-align: center; color: red;"> <p>↓</p> <p>$(x+4)(x+4) = 20$</p> </div>
5) Factor the trinomial on the left side	$(x+4)^2 = 20$
6) Take the square root of each side and solve (Remember that there are 2 solutions)	$x + 4 = \pm\sqrt{20}$ $x = -4 \pm \sqrt{20} = -4 \pm 2\sqrt{5}$ $x = -4 + 2\sqrt{5}$ $x = -4 - 2\sqrt{5}$

$$\begin{array}{c}
 -4 \pm \sqrt{20} \\
 -4 \pm 2\sqrt{5} \\
 \swarrow \quad \searrow \\
 -4 + 2\sqrt{5} \qquad -4 - 2\sqrt{5}
 \end{array}$$

Standard form

$$ax^2 + bx + c = 0$$

a, b, c are real numbers

6) Find the roots of the function $y = \frac{2}{2}x^2 - \frac{1}{2}x - \frac{1}{2}$

Recall that the roots of a function are on the x-axis where $y = 0$

zeros, solutions
roots all mean
to solve

$$\begin{array}{r} 0 = x^2 - \frac{1}{2}x - \frac{1}{2} \\ + \frac{1}{2} \qquad \qquad + \frac{1}{2} \\ \hline \frac{1}{2} = x^2 - \frac{1}{2}x \end{array}$$

$$x^2 - \frac{1}{2}x + \boxed{\frac{1}{16}} = \frac{1}{2} + \boxed{\frac{1}{16}}$$

Half it = $\frac{-\frac{1}{2}}{2} = -\frac{1}{4}$

Square it = $(-\frac{1}{4})^2 = \frac{1}{16}$

Share it = $\frac{1}{16}$

$$x^2 - \frac{1}{2}x + \boxed{\frac{1}{16}} = \frac{1}{2} + \boxed{\frac{1}{16}}$$

$$\left(x - \frac{1}{4}\right)^2 = \frac{9}{16}$$

Always
be the
"Half it"
number

$$\pm \sqrt{\left(x - \frac{1}{4}\right)^2} = \pm \sqrt{\frac{9}{16}}$$

$$\frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$$

$$x - \frac{1}{4} = \pm \frac{3}{4}$$

$$x = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$

$$x = \frac{1}{4} - \frac{3}{4} = -\frac{2}{4} = -\frac{1}{2}$$

$$x = \frac{1}{4} - \frac{3}{4} = -\frac{2}{4} = -\frac{1}{2}$$

$$x = -\frac{1}{2}, 1$$

$$1) \quad v^2 + 10v - 21 = 0$$

$+21 \quad +21$

$$v^2 + 10v + 25 = 21 + 25$$

← here

$$\text{Half} = \frac{10}{2} = 5$$
$$\text{Square} = 5^2 = 25$$

$$(v+5)^2 = 46$$

$$v+5 = \pm \sqrt{46}$$

$$\underline{-5 \quad -5}$$

$$v = -5 \pm \sqrt{46}$$

$$18) \quad \frac{4n^2}{4} + \frac{4n}{4} = \frac{-24}{4}$$

$$n^2 + n + \boxed{\frac{1}{4}} = -6 + \boxed{\frac{1}{4}}$$

$$\frac{1}{2} \quad \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\left(n + \frac{1}{2}\right)^2 = \frac{-23}{4}$$

$$\sqrt{\frac{-23}{4}} = \frac{\sqrt{-23}}{\sqrt{4}} = \frac{\sqrt{-23}}{2}$$

$$n + \frac{1}{2} = \pm \sqrt{\frac{-23}{4}}$$

$$n = -\frac{1}{2} \pm \frac{\sqrt{-23}}{2}$$

$$n = -\frac{1}{2} \pm \frac{i\sqrt{23}}{2}$$

$$2) \quad b^2 - 4b - 12 = 0$$

$$b^2 - 4b + \boxed{4} = 12 + \boxed{4}$$

$$\frac{-4}{2} = -2$$

$$(-2)^2 = 4$$

$$(b-2)^2 = 16$$

$$b-2 = \pm\sqrt{16}$$

$$b-2 = \pm 4$$

$$b = 2 \pm 4$$

$$b = 2 + 4 = 6$$

$$b = 2 - 4 = -2$$

7) Solve for x by completing the square:

$$x^2 + 2x = 4$$

$$x^2 + 2x + \boxed{1} = 4 + \boxed{1}$$

$$\frac{2}{2} = 1$$

$$1^2 = 1$$

$$\pm \sqrt{(x+1)^2} = \pm \sqrt{5}$$

$$x+1 = \pm \sqrt{5}$$

$$-1 = -1$$

$$x = \pm \sqrt{5} - 1$$

$$x = -1 \pm \sqrt{5}$$

HW Sheet

$$\#1) \quad v^2 + 10v - 21 = 0$$
$$\quad \quad \quad +21 \quad +21$$

$$v^2 + 10v + 25 = 21 + 25$$

$$\frac{10}{2} = 5$$

$$5^2 = 25$$

$$\sqrt{(v+5)^2} = \pm \sqrt{46}$$

$$v+5 = \pm \sqrt{46}$$

$$v = -5 \pm \sqrt{46}$$

$$2) \quad b^2 - 4b - 12 = 0$$

$$\quad \quad \quad +12 \quad +12$$

$$b^2 - 4b + \underline{4} = 12 + \underline{4}$$

$$\frac{-4}{2} = -2$$

$$(-2)^2 = 4$$

$$(b-2)^2 = 16$$

$$b-2 = \pm\sqrt{16}$$

$$b-2 = \pm 4$$

$$\quad \quad +2 \quad +2$$

$$b = \pm 4 + 2$$

$$b = +4 + 2$$
$$\textcircled{6}$$

$$\text{and } b = -4 + 2$$
$$\textcircled{-2}$$

$$3) \quad \begin{array}{r} v^2 - 14v - 44 = 0 \\ +44 \quad +44 \\ \hline \end{array}$$

$$\begin{array}{l} -14 \\ \hline 2 = -7 \end{array} \quad v^2 - 14v + \underline{49} = 44 + \underline{49}$$

$$\begin{array}{l} (-7)^2 = 49 \\ (v-7)^2 = 93 \end{array}$$

$$\begin{array}{r} v-7 = \pm \sqrt{93} \\ +7 \quad +7 \\ \hline \end{array}$$
$$v = 7 \pm \sqrt{93}$$

$$\begin{array}{r} 11) \quad 9n^2 = -77 + 18n \\ \quad \quad -18n \quad \quad -18n \\ \hline \frac{9n^2 - 18n}{9} = \frac{-77}{9} \end{array}$$

$$n^2 - 2n + \underline{\quad} = \frac{-77}{9} + \underline{\quad}$$

$$\frac{-2}{2} = -1$$

$$(-1)^2 = 1$$

$$\sqrt{(n-1)^2} = \pm \sqrt{\frac{-68}{9}}$$

$$n-1 = \pm \sqrt{\frac{-68}{9}}$$

$$n-1 = \pm \frac{2i\sqrt{17}}{3}$$

$$n = 1 \pm \frac{2i\sqrt{17}}{3}$$

$$\begin{aligned} \sqrt{\frac{-68}{9}} &= \frac{\sqrt{-68}}{\sqrt{9}} \\ &= \frac{i2\sqrt{17}}{3} \end{aligned}$$

$$\sqrt{\frac{-68}{9}} = \frac{\sqrt{-68}}{\sqrt{9}} = \frac{\sqrt{-68}}{(3)}$$

$$\begin{array}{c} \sqrt{-68} \\ \swarrow \searrow \\ \sqrt{-4} \quad \sqrt{17} \end{array}$$

$$\frac{2i\sqrt{17}}{3}$$



$$\begin{array}{r}
 12) \quad 9v^2 + 5 = 18v \\
 \quad \quad -18v \quad -18v \\
 \hline
 9v^2 - 18v + 5 = 0 \\
 \quad \quad \quad -5 \quad -5 \\
 \hline
 9v^2 - 18v = -5 \\
 \frac{9v^2}{9} \quad \frac{-18v}{9} = \frac{-5}{9}
 \end{array}$$

$$\begin{array}{l}
 -\frac{2}{2} = -1 \\
 (-1)^2 = 1
 \end{array}
 \quad
 \begin{array}{l}
 v^2 - 2v + \underline{\quad} = -\frac{5}{9} + \underline{\quad} \\
 \sqrt{(v-1)^2} = \pm \sqrt{\frac{4}{9}}
 \end{array}$$

$$\begin{array}{r}
 v - 1 = \pm \frac{2}{3} \\
 +1 \quad +1 \\
 \hline
 v = 1 \pm \frac{2}{3}
 \end{array}$$

$$v = 1 + \frac{2}{3} = \left(\frac{5}{3}\right)$$

$$v = 1 - \frac{2}{3} = \left(\frac{1}{3}\right)$$

$$v = \frac{1}{3}, \frac{5}{3}$$

$$15) \quad \frac{9m^2}{9} - \frac{20m}{9} - \frac{21}{9} = \frac{0}{9}$$

$$m^2 - \frac{20}{9}m - \frac{21}{9} = 0$$

$$+ \frac{21}{9} \quad + \frac{21}{9}$$

$$\frac{-\frac{20}{9}}{2} = -\frac{10}{9} \quad m^2 - \frac{20}{9}m + \frac{100}{81} = \frac{21}{9} + \frac{100}{81}$$

$$\left(-\frac{10}{9}\right)^2 = \frac{100}{81}$$

$$\sqrt{\left(m - \frac{10}{9}\right)^2} = \pm \sqrt{\frac{289}{81}}$$

$$m - \frac{10}{9} = \pm \frac{17}{9}$$

$$+ \frac{10}{9} \quad + \frac{10}{9}$$

$$m = \frac{10}{9} \pm \frac{17}{9}$$

$$m = \frac{10}{9} + \frac{17}{9} = \frac{27}{9}$$

$$m = 3$$

$$m = \frac{10}{9} - \frac{17}{9} = -\frac{7}{9}$$

$$m = -\frac{7}{9}$$

$$16) \quad \frac{10x^2}{10} - \frac{4x}{10} - \frac{32}{10} = \frac{0}{10}$$

$$x^2 - \frac{4}{10}x - \frac{32}{10} = 0$$

$$x^2 - \frac{2}{5}x - \frac{16}{5} = 0$$

$$\left. \begin{array}{l} \frac{-\frac{2}{5}}{2} = -\frac{1}{5} \\ \left(-\frac{1}{5}\right)^2 = \frac{1}{25} \end{array} \right\} x^2 - \frac{2}{5}x + \frac{1}{25} = \frac{16}{5} + \frac{1}{25}$$

$$\sqrt{\left(x - \frac{1}{5}\right)^2} = \pm \sqrt{\frac{81}{25}}$$

$$\begin{array}{rcl} x - \frac{1}{5} & = & \pm \frac{9}{5} \\ + \frac{1}{5} & & + \frac{1}{5} \\ \hline x & = & \frac{1}{5} \pm \frac{9}{5} \end{array}$$

$$x = \frac{1}{5} + \frac{9}{5} = 2 \quad x = \frac{1}{5} - \frac{9}{5} = -\frac{8}{5}$$